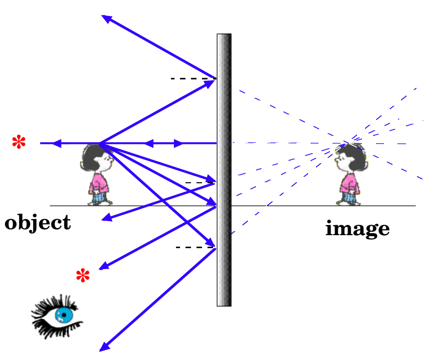
CHAPTER 32

OPTICAL IMAGES

- **Introduction to ray diagrams**
- **Reflection from mirrors**
  - † ***Plane mirrors***
  - † ***Spherical mirrors***
  - † ***Sign convention***
- **Refraction from curved surfaces**
- **Lenses**
  - † ***Converging Lenses***
  - † ***Diverging Lenses***
  - † ***Chromatic aberration***

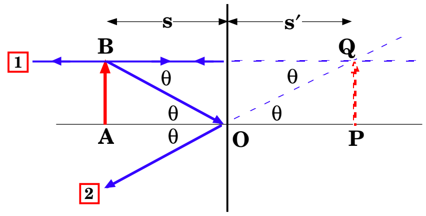
Introduction to ray diagrams:



Rays emerge from <u>all points</u> on an object. Only *some* of those from the top of the head are shown here. Note: the image *does not really exist* ... you cannot display it on a screen, for example!! It is called a *virtual image*.

***** Note that only 2 rays are required to locate the image position *****

Geometry of image formation (plane mirror):



ΔOAB and ΔOPQ are *congruent* triangles,
i.e.,

$$OA = OP$$
$$AB = PQ$$

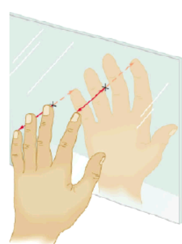
Lateral magnification $m = \frac{PQ}{AB}$ **(= 1).**

Also,

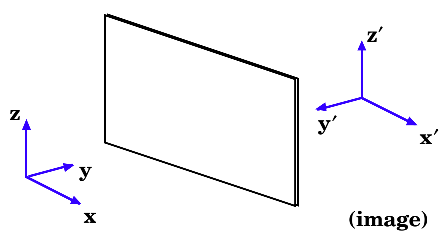
object distance **(OA) =** ***image distance*** **(OP)**
i.e.,

$$s = s'$$

Image is *virtual* and *upright*

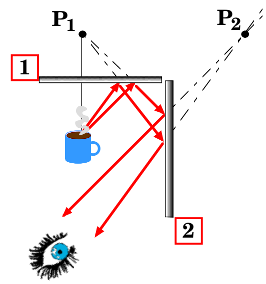


Reflection in a plane mirror produces:
lateral inversion



RH set of axes
(object)

Formation of an image with two plane mirrors.



The rays from the object into the eye satisfy the law of reflection. We see that the image at P_1 due to mirror 1 acts as an object for mirror 2. The final image appears at P_2. Note that both images are virtual.

***IMPORTANT CONCEPT*: Even though the image at P_1 is *virtual*, it can act as an object for mirror 2.**

DISCUSSION QUESTION [32.1]:

When the hairdresser shows you the back of your head by using a second mirror, is the image you see "laterally inverted" or "normal"?

DISCUSSION QUESTION [32.2]:

Can you photograph a virtual image?

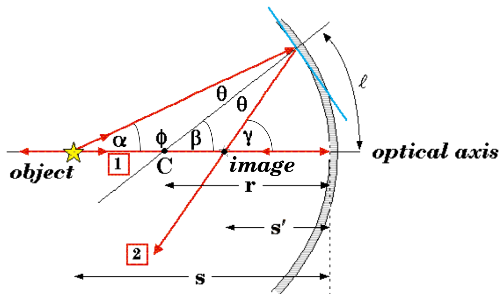
See the notes I wrote on

"retroreflection"

available on the useful notes link on the web-site.

Reflection from curved surfaces ...

Concave mirror:



Using simple geometry we have:

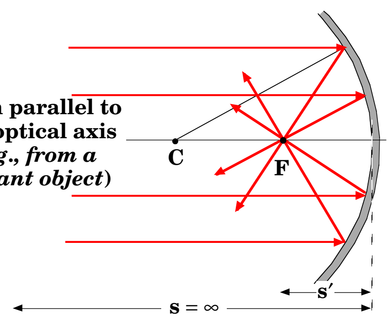
$\beta = \alpha + \theta$ **and** $\gamma = \alpha + 2\theta$.

Eliminate θ **and we get:** $\alpha + \gamma = 2\beta$

If α, β **and** γ **are small then:**

$$\alpha \approx \frac{\ell}{s} \qquad \beta \approx \frac{\ell}{r} \qquad \gamma \approx \frac{\ell}{s'}$$

i.e., $$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$



$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$$

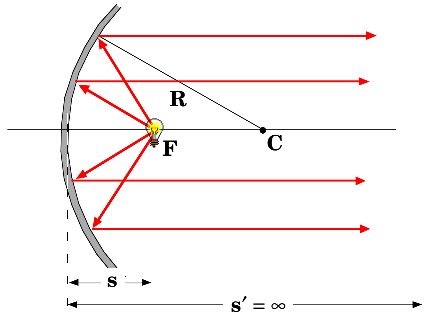
If $s = \infty$: $$s' = \frac{r}{2} = f,$$

where F is the ***principal focus*** **and f is the** ***focal length***.

$$\therefore \quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$$

Note, it is a ***real focus***.

EXAMPLES ?



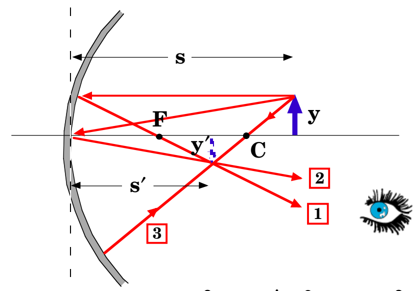
To produce a parallel beam of light put the source at the *principal focus*.

EXAMPLES ?

- ***Object distance*** **(s): when the object is on the *same side* of the reflecting or refracting surface as the *incoming light*, s > 0.**

- ***Image distance*** **(s′): when the image is on the *same side* of the reflecting or refracting surface as the *outgoing light*, s′ > 0.**

- ***Radius of curvature*** **(r): when the center of curvature is on the *same side* as the *outgoing light*, r > 0.**



$s > 0 \qquad s' > 0 \qquad r > 0$

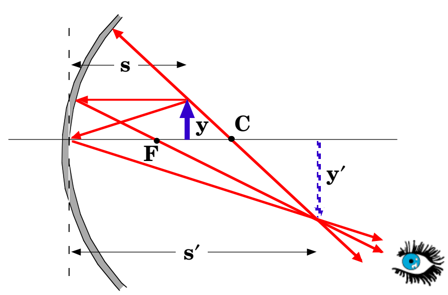
Object distance: s > r

The *lateral* (or *linear*) magnification is defined as: $m = \frac{y'}{y}$**. But** $\frac{y}{s} = -\frac{y'}{s}$**,**

$$\therefore m = -\frac{s'}{s} \qquad (\Rightarrow \text{negative})$$

Definition of magnification

Note in this case that $m < 0$ **and** $|m| < 1$**. The image is *real*, *inverted* and *reduced*.**



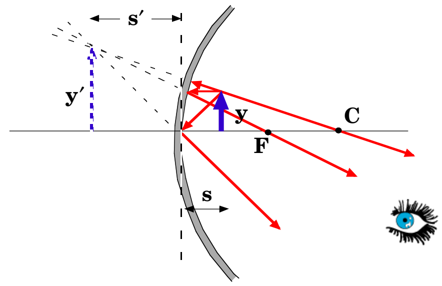
$s > 0 \qquad s' > 0 \qquad r > 0$

Object distance: $\frac{r}{2} < s < r$

Again, the lateral (linear) magnification is:

$$m = \frac{y'}{y} = -\frac{s'}{s} \qquad (\Rightarrow \text{negative}).$$

Note that $|m| > 1$ **and the image is *real*, *inverted* and *enlarged*.**



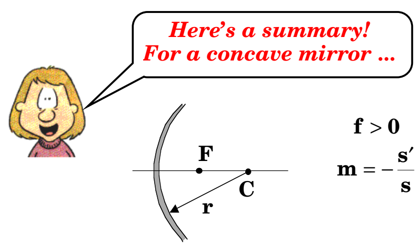
s > 0 s′ < 0 r > 0

Object distance: $s < \frac{r}{2}$, i.e., $s < f$

The lateral magnification is:

$$m = \frac{y'}{y} = -\frac{s'}{s} \quad (\Rightarrow \text{positive})$$

Note that $|m| > 1$ and the image is *upright*.



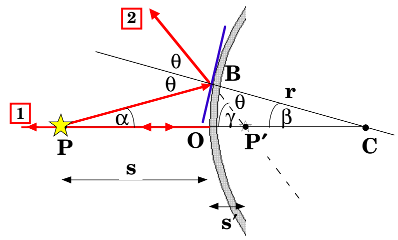
$f > 0$

$m = -\frac{s'}{s}$

- **When $s > r$: then $m < 0$ and $|m| < 1$. The image is *real*, *inverted* and *reduced*.**
- **When $f < s < r$: then $m < 0$ and $|m| > 1$. The image is *real*, *inverted* and *enlarged*.**
- **When $s < f$: then $m > 0$ and $|m| > 1$. The image is *virtual*, *upright* and *enlarged*.**

positive m ($m > 0$) ⇔ virtual image (upright)
negative m ($m < 0$) ⇔ real image (inverted)

Convex mirror:



Using simple geometry:

$$\theta = \alpha + \beta \quad \textbf{and} \quad 2\theta = \alpha + \gamma$$

Eliminate θ and we get: $\gamma - \alpha = 2\beta$

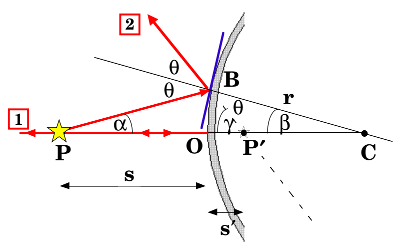
i.e., $\frac{\ell}{s'} - \frac{\ell}{s} = \frac{2\ell}{r}$ **so,** $\frac{1}{s'} - \frac{1}{s} = \frac{2}{r}.$

But, using the *sign convention* we find:

$$s > 0 \qquad s' < 0 \qquad r < 0,$$

$$\therefore \frac{1}{s'} + \frac{1}{s} = \frac{2}{r},$$

which is the same result we obtained for a concave mirror!!



So, using the sign convention, we find ...

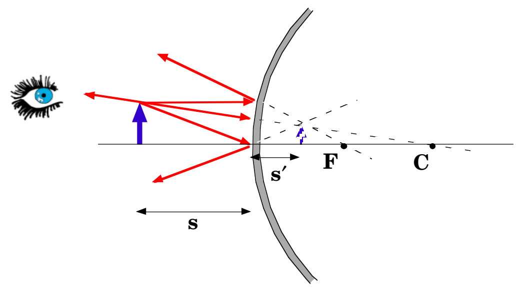
$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{r}.$$

When $s = \infty$**, i.e., an incoming parallel beam of light, we have:** $\frac{1}{s'} = \frac{2}{r} = \frac{1}{f}$

i.e., $f = \frac{r}{2}$ **(*virtual* focus),**

so $\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}.$

**** Note that by the sign convention, r < 0, and so f < 0 for a convex lens.**



By simple geometry the lateral magnification of a convex mirror is:

$$m = \frac{y'}{y} = -\frac{s'}{s} \quad (\text{but } s' < 0, \text{ so } m > 0)$$

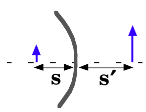
Note: m is *always* < 1 and the image is *always* *upright* and *virtual* and *reduced in size*.

SIMPLE EXAMPLES **?**

PROBLEM 32.35 page 1078 :

(b) The mirror cannot be convex because a convex mirror *always* produces a *diminished* virtual image. Therefore, it must be concave.

(a)



s = 0.021m. The image is upright and so it is *virtual*, i.e., behind the mirror, so s′ < 0.

The magnification: $m = -\frac{s'}{s} = 5.5$

$$\therefore s' = -5.5 \times 0.021 = -0.116\,\text{m}.$$

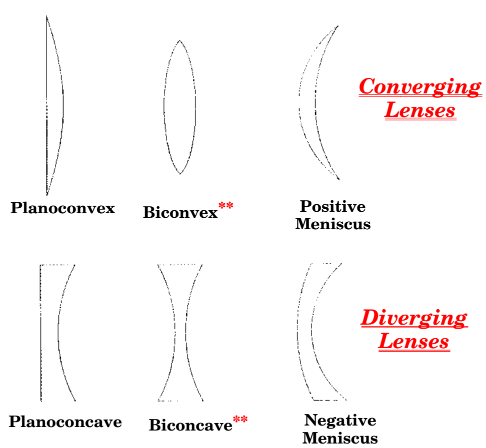
$$\therefore \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{2}{r} = \frac{1}{0.021} - \frac{1}{0.116} = 39.0$$

$$\text{i.e.,} \quad r = \frac{2}{39.0} = 0.051\text{m} \ \ (5.1\text{cm})$$

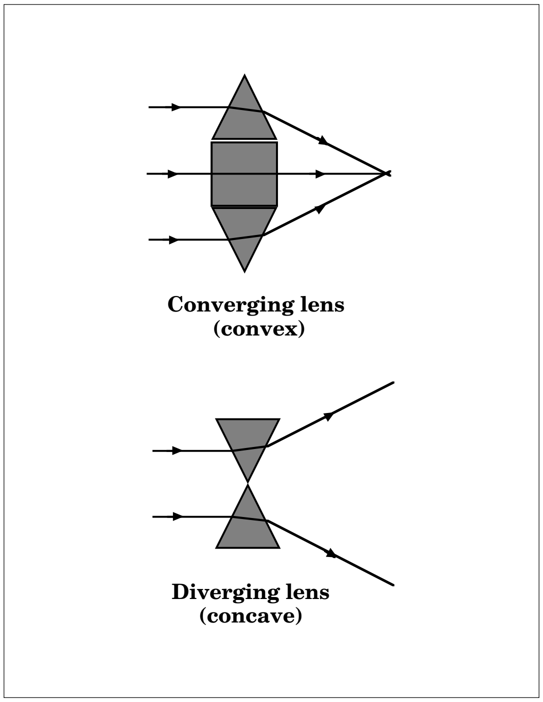
$$\therefore f = 2.55\text{cm, so } s < f.$$

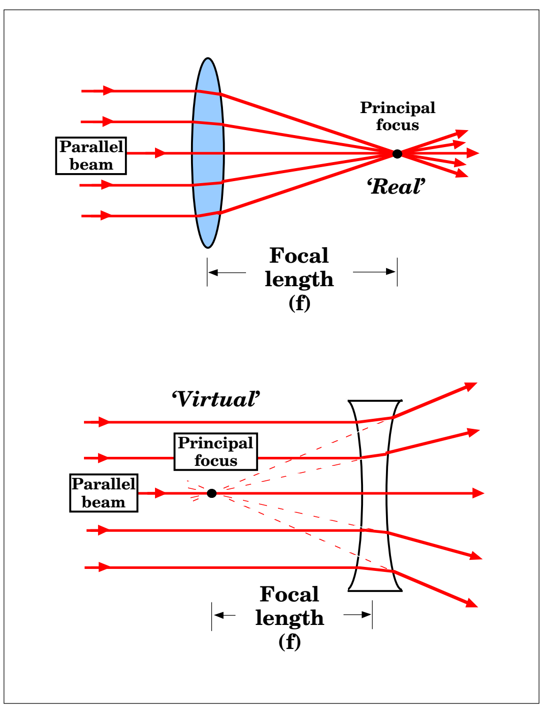
See the notes on the web-site for chapter 32 on how to solve mirror problems using a scale drawing ...

Lenses ...

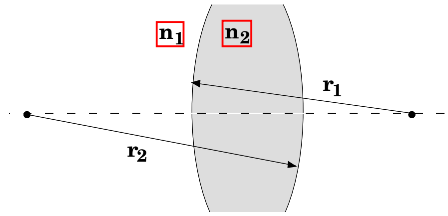


Cross-sections of various types of lenses. The top group are *converging* lenses, the lower group are *diverging* lenses.


Converging lens
(convex)
Diverging lens
(concave)


Principal
focus
Parallel
beam
'Real'
Focal
length
(f)
'Virtual'
Principal
focus
Parallel
beam
Focal
length
(f)

Lens-maker's formula :

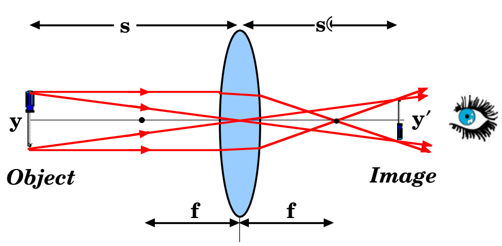


$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Optometrists who prescribe lenses and opticians who make them usually do not specify the focal length but the *refractive power*, in units called *diopters*.

Refractive power $= \dfrac{1}{\text{f (in meters)}}$ **(diopters)**

Converging lenses have $f > 0$ but diverging lenses have $f < 0$ (i.e., a negative number).

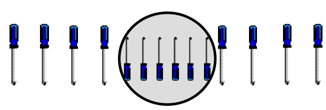


For a convex lens we have (see "useful notes"):

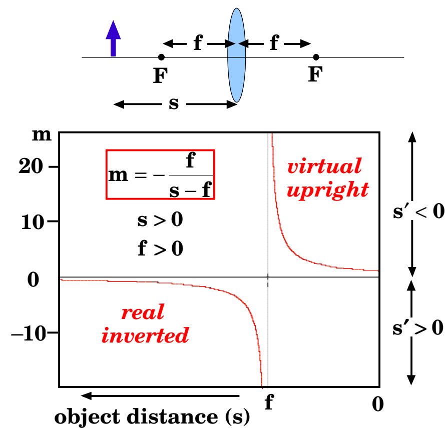
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

and $\quad m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

(Also, by substitution: $m = -\dfrac{s' - f}{f} = -\dfrac{f}{s - f}$**)**



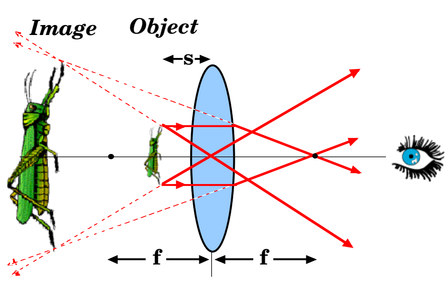
If $s > 2f$ then $m < 1$, i.e., reduced image.



When:

- s > f then m < 0 (***real, inverted***)
- s = 2f then |m| = 1 (***real, inverted***)
- s = f then m = ∞
- s < f then m > 1 (***virtual, upright***)

Interesting case !!



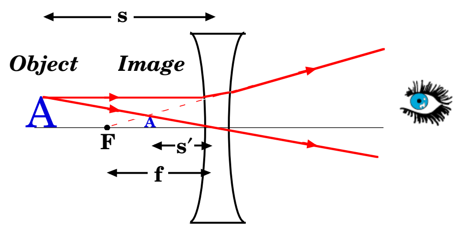
$$m = -\frac{f}{s-f}$$

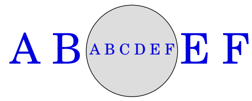
If s < f, then m > 1. Also,

since $\frac{1}{s} > \frac{1}{f}$, **then** $\frac{1}{s'} < 0$,

i.e., $s' < 0$.

So, to use a convex lens as a magnifying glass make s < f, i.e., object distance less than the focal length of the lens. The image is on the same side as the object and it is *enlarged*, *upright* and *virtual*.



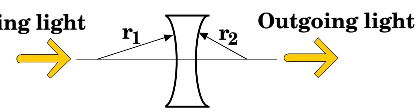


For a concave lens: $s > 0$, $s' < 0$ **and** $f < 0$.

Since $m = -\frac{s'}{s}$ **then** $1 > m > 0$, **always.**

So, a diverging lens *always* produces a *virtual image* of a real object no matter where it is positioned. The image is *upright* and *smaller* relative to the object

Problem 32.53, page 1079:



$r_1 = -0.30\text{m}$ (***opposite side of outgoing light***).

$r_2 = 0.25\text{m}$ (***same side of outgoing light***).

$$n_1 = 1.00 \quad \text{and} \quad n_2 = 1.45$$

(a)
$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \left(\frac{1.45}{1.00} - 1\right)\left(\frac{1}{-0.30} - \frac{1}{0.25}\right) = -3.3$$

$$\therefore f = -0.303\text{m} \quad (\textit{diverging lens}).$$

(b) $s = 0.80\text{m}$ **and** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

So, $\frac{1}{0.8} + \frac{1}{s'} = \frac{1}{-0.303}$ **i.e.,** $\frac{1}{s'} = \frac{1}{-0.303} - \frac{1}{0.8}$

$$\therefore s' = -0.22\text{m}$$

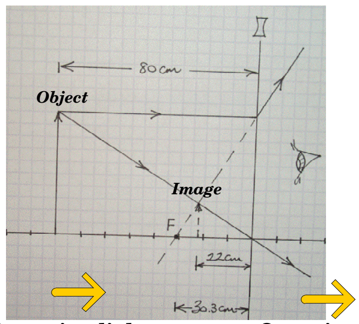
(***opposite side to outgoing light***)

Problem 32.53, page 1079 (continued):

(c) $m = -\frac{s'}{s} = \frac{0.22}{0.8} = 0.275.$

(d) Since $m > 0$ the image is *virtual* and *upright*.

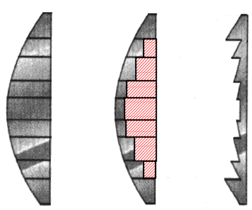
See useful notes on web-site to see how to solve this problem using a scale drawing.



Incoming light Outgoing light

Note: the image is not necessarily on the same side as the outgoing light!!

A Fresnel Lens:



Only the radii of the front and back surfaces of a lens determine the focal length, not the thickness in between! So, much weight can be saved with this plano-convex lens by removing the "unused" part of a lens!

A Fresnel (biconvex) lens

Lighthouse lens