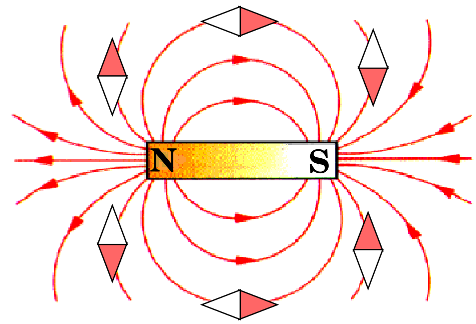
CHAPTER 26

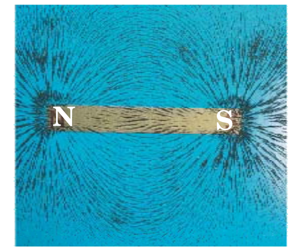
THE MAGNETIC FIELD

- **Force exerted on a charge by a magnetic field**
- **Motion of a charge in a magnetic field**
  - † *Cyclotron*
  - † *Velocity selector*
- **Torques on current loops and coils**
  - † *Magnetic moment*

Examples of magnetic of magnetic fields ...

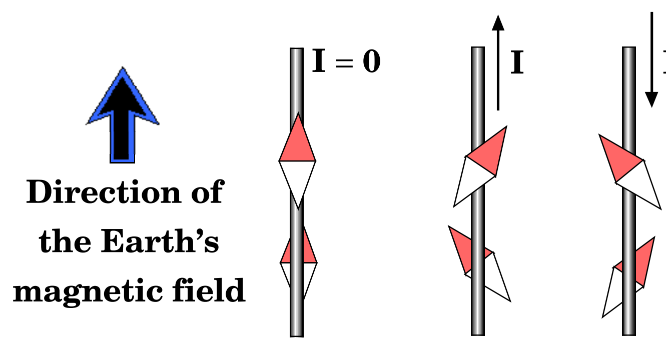
(indicated by compass needles or iron filings)





Man made or naturally occuring ores ...

permanent magnet



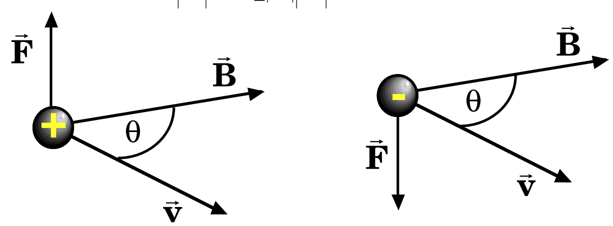
Oersted's experiment (1819)...

electric current

Force on a moving charge:

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$|\vec{F}| = q|\vec{v}||\vec{B}|\sin\theta$$



- **Direction of $\vec{F}$ depends on sign of q.**
- **Charge must be moving!**
- **If $\theta = 0$ then $|\vec{F}| = 0$.**
- **Motion continues at constant speed ...**

... ***Why??***

- **Direction of $\vec{F}$ given by the right-hand rule for a positive charge ...** ***next slide!***

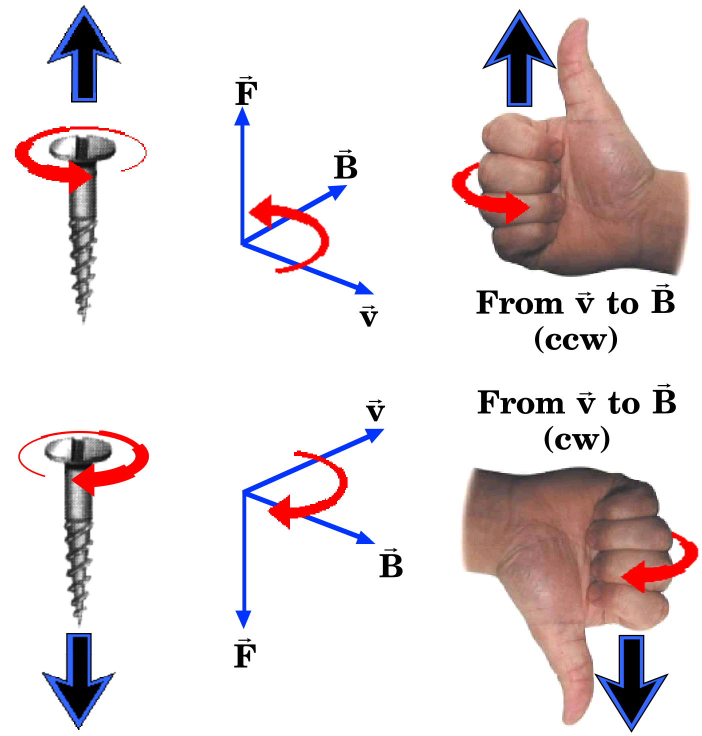
UNITS: $\vec{F} \Rightarrow N$ $q \Rightarrow C$ $\vec{v} \Rightarrow m/s$

$$\vec{B} \Rightarrow \text{Tesla (T)}$$

$$1 \text{ Gauss (G)} = 10^{-4}\text{T}$$

The Right-Hand Rule...

$$\vec{F} = q\vec{v} \times \vec{B}$$



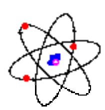
The right-hand-rule gives the direction of $\vec{F}$ ... true for all vector products.

Note that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$.

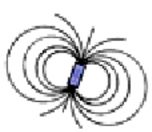
The Earth's magnetic field ... ~0.6G

Magnets used to "stick" papers to a refrigerator ... ~100G

The magnetic field associated with sunspots ... ~1000G



The magnetic field inside atoms ... ~100,000G (10T)

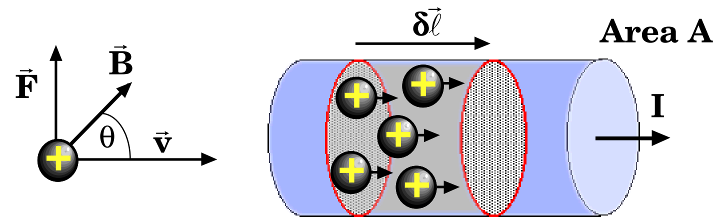


The strongest sustained magnetic fields produced by scientists ... ~450,000G (45T).
Pulsed field ~150T.

Neutron stars ... ~10^8T

Magnetars ... ~10^{11}T

Force on a straight wire in a magnetic field:



Force on each moving charge $\Rightarrow q\vec{v}\times\vec{B}$.
Since the charges are "held" in the wire, the total force on the element of wire length $|\delta\ell|$ is:

$$\delta\vec{F} = nA|\delta\vec{\ell}|q\vec{v}\times\vec{B},$$

where n is the number of charges per unit volume and $A|\delta\vec{\ell}|$ is the volume element. But, from Ch. 25, the current in the wire is:

$$I = nqA|v|.$$

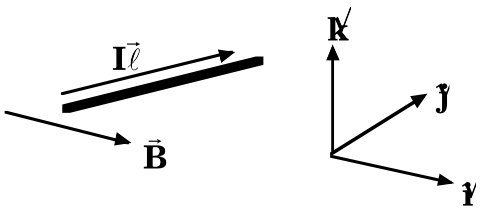
If $\delta\vec{\ell}$ is parallel to $\vec{v}$, i.e., the current direction, then:

$$\boxed{\delta\vec{F} = I\delta\vec{\ell}\times\vec{B}}$$

So, for a straight wire of length $\vec{\ell}$, the total force is:

$$|\vec{F}| = I|\vec{\ell}||\vec{B}|\sin\theta$$

Problem 26.22 page 851:



We know:

$$\vec{F} = I\vec{\ell} \times \vec{B},$$

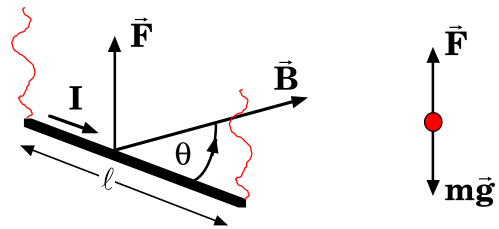
i.e., $\vec{F} = 2.7(0.03\hat{i} + 0.04\hat{j}) \times 1.3\hat{i}$

$= 0.1053\hat{i} \times \hat{i} + 0.1404\hat{j} \times \hat{i}$

But $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{i} = -\hat{k}$ (*remember??*)

$$\therefore \vec{F} = -0.1404\hat{k}\ \text{N}.$$

Problem 26.25 page 851:



To "float", the net force on the wire must be 0.

i.e., $|\vec{F}| = |m\vec{g}|.$

$$\therefore F = I|\vec{\ell}||\vec{B}|\sin\theta = mg,$$

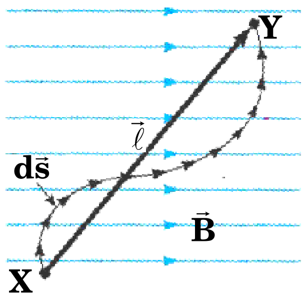
i.e., $I = \dfrac{mg}{|\vec{\ell}||\vec{B}|\sin\theta}.$

$\ell = 0.25\text{m}, \quad B = 1.33\text{T}, \quad \theta = 90^\circ, \quad m = 0.05\text{kg},$

$g = 9.81\text{m/s}^2.$

$$\therefore I = \frac{0.05 \times 9.81}{0.25 \times 1.33} = 1.47\text{A}.$$

What's the force if the wire isn't straight?



Break the wire XY into small elements ($d\vec{s}$). If the wire carries a current I, the force on an element $d\vec{s}$ is:

$$d\vec{F} = I d\vec{s} \times \vec{B}.$$

$\therefore$ Total force on wire is: $\vec{F} = \int_X^Y I d\vec{s} \times \vec{B}.$

Since I and $\vec{B}$ are constant: $\vec{F} = I\left\{\int_X^Y d\vec{s}\right\} \times \vec{B}.$

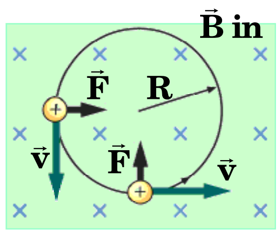
But, $\int_X^Y d\vec{s}$ is the vector sum of the elements from $X \rightarrow Y \Rightarrow \vec{\ell}$, i.e., the displacement $\overrightarrow{XY}$.

$$\therefore \vec{F} = I\vec{\ell} \times \vec{B}.$$

Special case: with a *closed* loop, $\vec{\ell} = 0$ so $F = 0$, i.e., a closed loop will experience no net force!

Charges in a magnetic field ...

Cyclotron:



The force on the charge is

$$|\vec{F}| = q|\vec{v}||\vec{B}|.$$

and it is always "inwards". Since there is *no change in speed* (*why??*) this force is constant and acts as a *centripetal force*,

$|\vec{F}| = m\dfrac{v^2}{R}$, and produces circular motion.

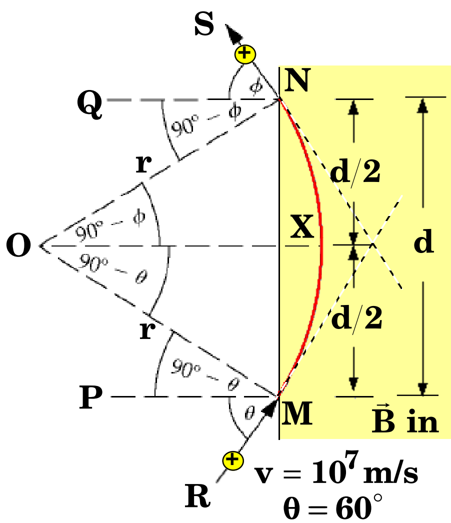
$$\therefore qvB = m\frac{v^2}{R}, \qquad \text{i.e., } R = \frac{mv}{qB}.$$

Note, with $\vec{B}$ *into* the page, the rotation is *ccw* for a *positive* charge. The time to orbit (the *cyclotron period*) is:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}.$$

The *cyclotron frequency* is: $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}.$

Problem 26.38 page 852:



RM and NS are straight lines (no field → no force); M to N (*in the field*) is an arc of a circle, with radius r, and its center at O.

$$\angle RMO = \angle SNO = 90^\circ \qquad \text{(radius/tangent)}$$

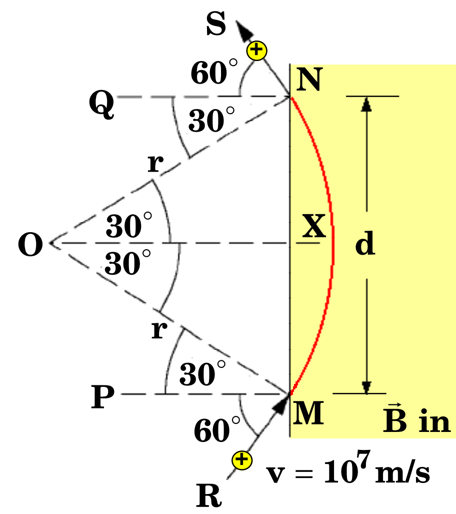
$$\angle PMO = 30^\circ = \angle MOX \qquad \text{(alternate angle)}$$

By symmetry $\angle NOX = 30^\circ = 90^\circ - \phi. \qquad \therefore \phi = 60^\circ.$

$$\text{Also,}\ \sin(90^\circ - \phi) = \sin 30^\circ = \frac{d/2}{r} = \frac{1}{2}$$

$$\therefore d = r$$

Problem 26.38 page 852 (continued):

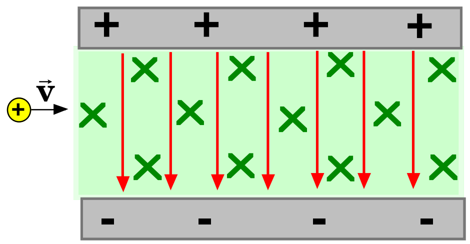


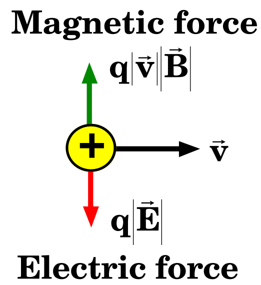
From earlier: $r = \dfrac{mv}{qB}$

$$= \frac{1.67 \times 10^{-27} \times 10^{7}}{1.6 \times 10^{-19} \times 0.8} = 0.1305\text{m}$$

$$\therefore d = 0.1305\ \text{m}$$

Velocity selector:





Magnetic field into the page
Electric field vertically down

The *total* force on the charge is:

$$\vec{F} = q\vec{E} \ + \ q\vec{v} \times \vec{B}$$

There is no net force ($|\vec{F}| = 0$) if $q|\vec{E}| = q|\vec{v}||\vec{B}|$,

$$\text{i.e., when } \quad |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|}.$$

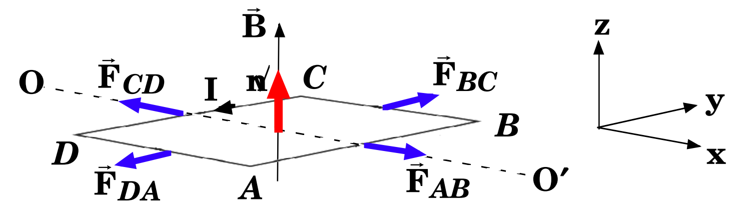
So, only charged particles with *that particular velocity* will pass through undeviated (no matter if charge is ±ve !).

If charge is +ve:

Faster ⇒ larger "magnetic" force

Slower ⇒ smaller "magnetic" force

Force on a coil carrying a current:



$AB = \ell_1$ and $BC = \ell_2$

1. Coil perpendicular to a magnetic field:

The current (I) in each side is the same and the angles between $I\vec{\ell}$ and $\vec{B}$ is 90°. Therefore,

$$F_{AB} = I\ell_1 B \qquad F_{BC} = I\ell_2 B$$

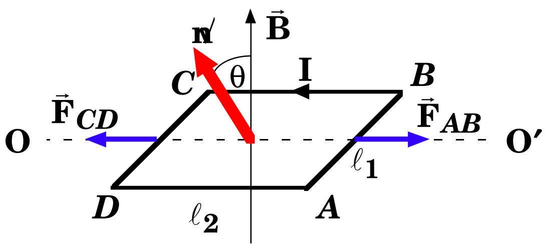
$$F_{CD} = -I\ell_1 B \qquad F_{DA} = -I\ell_2 B.$$

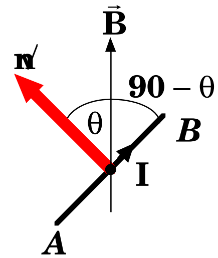
Since

$$F_{AB} = -F_{CD} \ \text{ and } \ F_{BC} = -F_{DA}$$

there is ***no net force on the coil***. (We could have predicted that ... how?) Also, since the forces are all in the same $(x - y)$ plane as the coil (by the RH rule), there is no net torque. The coil remains stationary in the magnetic field.

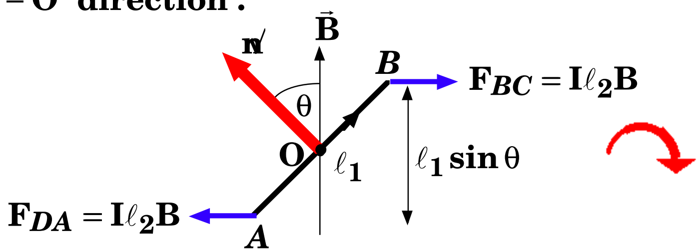
2. *Coil at an angle (θ) to a magnetic field*:





$F_{AB} = I\ell_1 B\sin(90-\theta)$ **and** $F_{CD} = I\ell_1 B\sin(270-\theta)$.

$\therefore \vec{F}_{AB} = -\vec{F}_{CD}$, **so there is no net force in the $O - O'$ direction .**

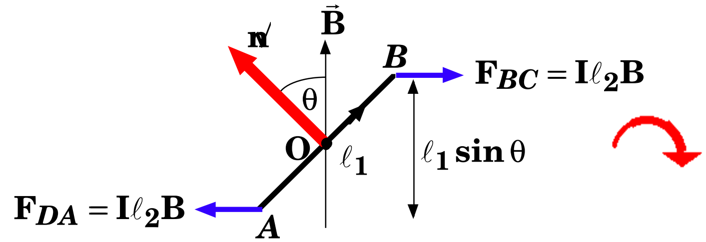


Note that F_{BC} and F_{DA} form a *couple* and produce a torque that tends to rotate the coil cw about $O - O'$. The total torque is:

$$\tau = (I\ell_2 B)\frac{1}{2}\ell_1\sin\theta + (I\ell_2 B)\frac{1}{2}\ell_1\sin\theta$$

$$= (I\ell_1\ell_2)B\sin\theta.$$

2. *Coil at an angle (θ) to a magnetic field*:

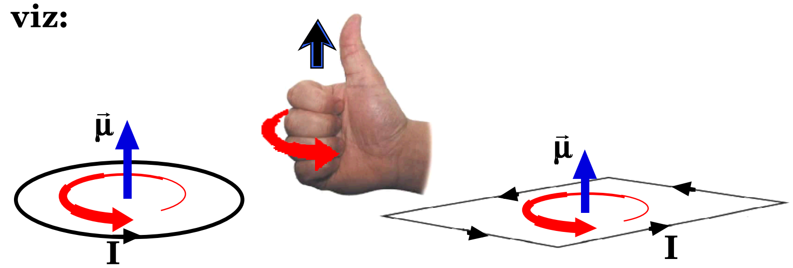


The torque: $\tau = (I\ell_1\ell_2)B\sin\theta = \mu B\sin\theta$,

area

where *we define*: $\mu = I\ell_1\ell_2 = IA$

as the *MAGNETIC MOMENT* of the coil (units $\Rightarrow A.m^2$). The magnetic moment is a vector in the n̂ direction given by the right hand rule, viz:



2. Coil at an angle (θ) to a magnetic field:

In general, the magnetic moment of a coil is:

$$\mu = IA.$$

For a coil with N turns of wire:

$$\mu = NIA.$$

So, the magnetic moment of a circular coil of radius R and N turns is:

$$\mu = NI\pi R^2,$$

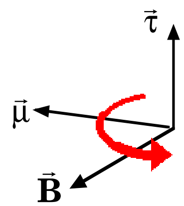
and if it is set an angle θ to a magnetic field $\vec{B}$, it experiences a torque:

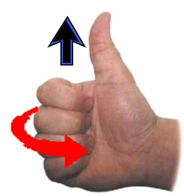
$$\tau = (NI\pi R^2)B\sin\theta.$$

NOTE: the ***direction*** of the torque is given by the RH rule. In vector notation:

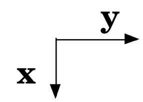
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\text{so } \tau = |\vec{\tau}| = \mu B \sin\theta$$



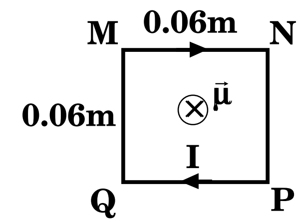


Problem 26.55 page 853 :



(a) $\vec{B}$ along z

(b) $\vec{B}$ along x



(a) From earlier: $\tau = \mu B \sin\theta$, with $\mu = IA$.

θ is the angle between $\vec{\mu}$ and $\vec{B}$ (= 0 or 180°).

$$\therefore \sin\theta = 0, \quad \text{i.e.,} \quad \tau = 0.$$

Note: $\mu = IA = 2.5 \times (0.06 \times 0.06) = 9.00 \times 10^{-3}\,\text{A.m}^2$.

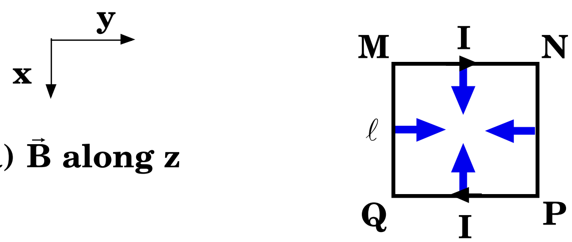
(b) $\theta = 90°$

$$\therefore \tau = \mu B \sin\theta = 9.00 \times 10^{-3} \times 0.3 \times 1$$

$$= 2.70 \times 10^{-3}\,\text{N.m.}$$

NOTE: The direction of $\vec{\tau}$ depends on the direction of $\vec{\mu}$, the magnetic moment of the coil. For the direction chosen, $\vec{\mu}$ is in the $-z$ direction and so $\vec{\tau}$ is in the $-y$ direction.

Problem 26.55 page 853 : (cont)



(a) $\vec{B}$ along z

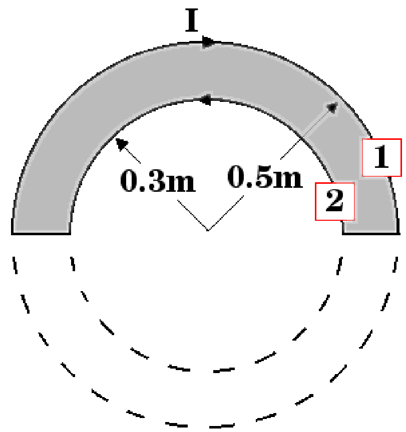
The force on each segment is:

$$F = I\ell B \sin\theta$$

where θ is angle between magnetic field and the current.

The direction of the forces is shown above; they have equal magnitudes and so the net force is zero. So, in (a) not only does the coil remain stationary, there is no torque either.

Problem 26.61 page 853 :



The magnetic moment is:

$$\mu = NIA$$

where A is the shaded area $= \frac{1}{2}\pi(R_1^2 - R_2^2)$.

$$\therefore \mu = \frac{1}{2} \times 1.5 \times \pi \times \left(0.5^2 - 0.3^2\right) = 0.377 A.m^2.$$

The direction of the (net) magnetic moment is ***inwards***.