I want you to study

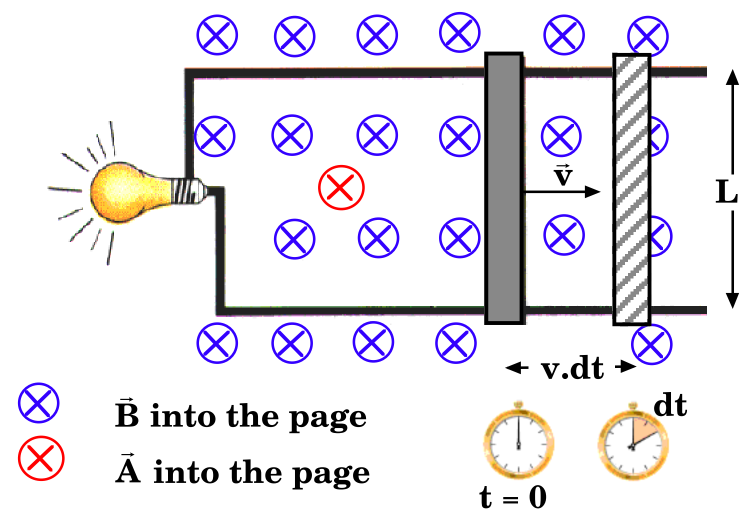
Motional emf

and

Energy density of a magnetic field

using these notes together with sections 28-4 and 28-7 of the textbook. There will be questions on these topics in the homework, on the quiz and on the test.

Motional emf - Slidewire generator:



$\vec{B}$ into the page

$\vec{A}$ into the page

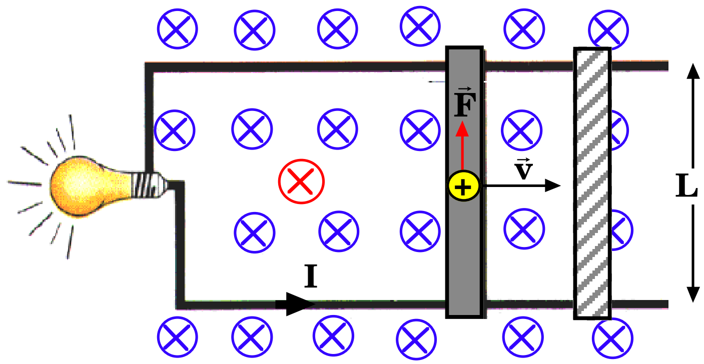
The change in flux (due to change in area):

$$d\Phi = B.dA = BLv.dt$$

$$\therefore \frac{d\Phi}{dt} = BLv,$$

so $\varepsilon = -BLv$.

- What are the directions of ε (and I)? (Can you think of 2 methods?)



[1] Force on charges: $\vec{F} = q\vec{v} \times \vec{B}$

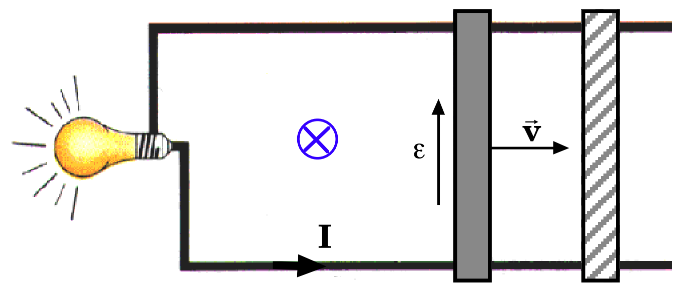
Direction/motion given by RH rule, i.e., current counter-clockwise

[2] Change of flux $\frac{d\Phi}{dt} > 0$,

$$\therefore \varepsilon = -\frac{d\Phi}{dt} < 0$$

i.e, with $\vec{A}$ directed inwards, ε (and I) are counter-clockwise.

<u>Slidewire generator</u> (continued):

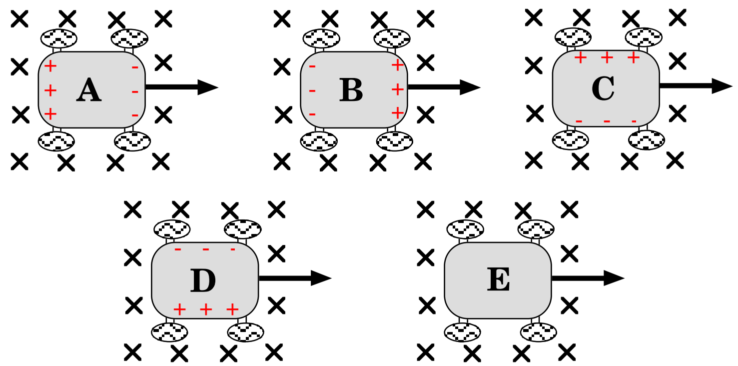


⊗ $\vec{B}$ into the page

From the conservation of energy ...

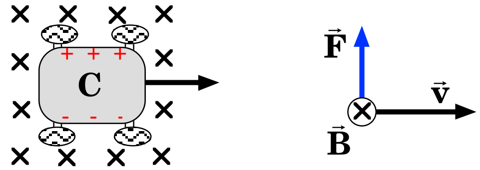
Assuming no friction, the mechanical work done in moving the slide against the field, *i.e., overcoming Lenz's Law (which opposes the change)*, is equal to the work done sending charges around the circuit (= εI).

Question [28.4]:



A metal cart is being pulled at constant speed across the North pole where the magnetic field is directed vertically downward. Assuming the field is uniform, which figure best indicates how charge is distributed across the bed of the cart?

Question [28.4]: ... answer C.



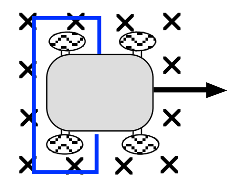
From notes: $\vec{F} = q\vec{v} \times \vec{B}$. Therefore, by the right hand rule, the force on positive charges is directed in the direction shown.

DISCUSSION PROBLEM [28.3]:

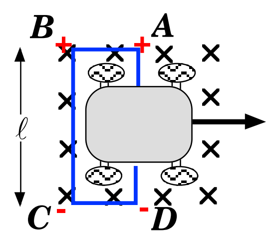
OK, the emf is very small, but could we use it to generate a current in a circuit?

DISCUSSION PROBLEM [28.3]: ... solution ...

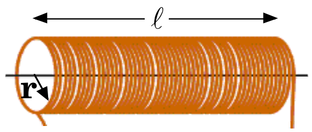
NO! **... because if we make a "circuit" there is no change in flux through the "loop" ($d\Phi/dt = 0$) and so, there is no induced emf ($\varepsilon = -d\Phi/dt$) to produce a current!**



Using a different argument, if the circuit loop is *ABCD*, the emf generated across *AD* ($\varepsilon = |B\ell v|$) is the same as that across *BC*. So there is no net emf in the circuit and therefore no current (even if *BC* is vertically above *AD*).



***Energy density of a magnetic field* ...**



Assume we have a solenoid of radius r, with n turns per unit length and length ℓ.

Then, from the notes, its inductance is:

$$L = \mu_\circ n^2 \pi r^2 \ell$$

$$\therefore U_L = \frac{1}{2} L I^2 = \frac{1}{2}(\mu_\circ n^2 \pi r^2 \ell) I^2 = \frac{1}{2}\frac{(\mu_\circ n I)^2}{\mu_\circ}\pi r^2 \ell.$$

But $\pi r^2 \ell \Rightarrow$ volume of solenoid, and $B = \mu_\circ n I$ is the magnetic field in the solenoid. So the *magnetic energy density* is:

$$U_L \big/ \text{volume} = \frac{1}{2}\frac{B^2}{\mu_\circ}.$$

This result is true for *all* magnetic fields.