CHAPTER 25

ELECTRIC CURRENT and DIRECT-CURRENT CIRCUITS

- Current as the motion of charges
  - *The Ampère*

- Resistance and Ohm’s Law
  - *Ohmic and non-Ohmic materials*

- Electrical energy and power

- Simple circuits

- Combinations of resistors
  - *Series and parallel*

- Kirchhoff’s Rules
  - *Example of an R-C circuit*

---

**ELECTRIC CURRENT**

Inside a conductor the charges are in constant, random motion. But generally, in metals, only the electrons are free to move, at speeds typically $\sim 10^6$ m/s. However, with no applied electric field the electrons move randomly:

Since there are many “free” electrons and their motions are uncorrelated, there is

*NO NET FLOW IN ANY DIRECTION.*
If a potential difference is applied, e.g., from a battery, a net flow of charge results ... 

**WHY?** Because an electric field is set-up and each electron experiences a force: \( \vec{F} = q\vec{E} \) towards the right ...

As a result, an electron that would have travelled from \( P_1 \rightarrow P_2 \), say, now travels from \( P_1 \rightarrow P_3 \). Indeed, all the electrons experience a force to the right and so there is a net flow (or “drift”) of −ve charge from left to right. The “drift velocity” of electrons is typically \( \sim 10^{-4} \text{ m/s} \).

The current is defined as the rate of flow of charge, i.e., the amount of charge passing through a given area each second, i.e., \( I = \frac{dQ}{dt} \).

The direction of the current is defined as the direction +ve charges would flow, i.e., in the same direction as \( \vec{E} \) (that is from higher potential to lower potential). In our example, even though electrons flow from left \( \rightarrow \) right, the current direction is from right \( \rightarrow \) left. Note: current is a BULK effect (i.e., not restricted to the surface).

**UNITS:** 
- \( dQ \Rightarrow \text{Coulombs (C)} \)
- \( dt \Rightarrow \text{seconds (s)} \)
- \( I \Rightarrow \frac{C}{s} \Rightarrow \text{Ampères (A)} \)
If the charges drift an average distance $\Delta x$ in a time $\Delta t$, the *drift velocity* $v_d$ is:

$$v_d = \frac{\Delta x}{\Delta t}, \text{ i.e., } \Delta x = v_d \Delta t.$$  

The number of charges in the shaded volume is:

$$n(A.\Delta x)$$

where $n$ is the number of charges per unit volume. In time $\Delta t$ the amount of charge, $\Delta Q$, that flows out of (and into) the volume is:

$$\Delta Q = n(A.\Delta x)q = nqA v_d \Delta t,$$

so the current is:

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA v_d \Delta t}{\Delta t} = nqA v_d.$$  

If the current is due only to electrons (normal in metals):

$$I = n|e|A|v_d|.$$  

When there are two types of charges, e.g., in a fluorescent tube, the *total* current is:

$$I = A \sum_i n_i q_i v_i \Rightarrow$$

$$= n_1 q_1 A v_{d1} + n_2 q_2 A v_{d2},$$

i.e., it is the sum of the currents due to each type of charge.

**NOTE:**

- if $q_1$ is positive then $v_{d1} > 0$, i.e., in the same direction as the current, so $q_1 v_{d1} > 0$.
- if $q_2$ is negative then $v_{d2} < 0$, i.e., in the opposite direction to the current, so $q_2 v_{d2} > 0$.

Therefore, the individual currents still *add*.  

**Question 25.1:** In a certain electrical device, $+1\text{C}$ of charge flows from left to right and $-5\text{C}$ of charge flows from right to left each second.

What is the net current, $I$?

A current of $-5\text{ C/s}$ to the *left* is equivalent to a current of $+5\text{ C/s}$ to the *right*, so the net current is

$$+1\text{ C/s} + 5\text{ C/s} = 6\text{ C/s} = 6\text{A}.$$
**OHM’s LAW** (1827)

The potential difference between sections a and b is:

\[ V_{ab} = V_a - V_b = E\ell \quad (V_a > V_b) \]

Georg Simon Ohm found that the ratio \( \frac{V_{ab}}{I} \) was constant. The constant is called the *resistance* of that segment of wire,

\[ \text{i.e.,} \quad \frac{V_{ab}}{I} = R. \]

Equivalent expressions for Ohm’s Law are:

\[ V_{ab} = IR \quad \text{and} \quad I = \frac{V_{ab}}{R}. \]

**UNITS:**

- \( V \Rightarrow \text{Volts (V)} \)
- \( I \Rightarrow \text{Ampères (A)} \)
- then \( R \Rightarrow \text{Ohms (Ω)} \)

**Specific resistance (also known as the resistivity):**

The actual *resistance* of the section depends on the length \( \ell \), the cross sectional area \( A \) and the type of material:

\[ R = \rho \frac{\ell}{A}, \]

where \( \rho \) is called the *specific resistance* or *resistivity* of the material.

**Typical values of \( \rho \) (in Ω.m):**

- Silver \( 1.6 \times 10^{-8} \)
- Copper \( 1.7 \times 10^{-8} \)
- Tungsten \( 5.5 \times 10^{-8} \)
- Nichrome \( 100 \times 10^{-8} \)
- Carbon \( 3500 \times 10^{-8} \)
- Glass \( 10^{10} \rightarrow 10^{14} \)
- Rubber \( 10^{13} \rightarrow 10^{16} \)
The resistivity $\rho$ of a material generally varies with temperature:

A: typical metal
B: typical semiconductor
C: typical superconductor

Consequently, Ohm’s Law is only valid provided the temperature of the material is constant.

However, not all materials and/or devices are “Ohmic”, i.e., obey Ohm’s Law; look at the semiconductor diode below.
Work done and power:

Consider a charge $\delta q$ moving under the influence of the electric field from $a \rightarrow b$ in a time $\delta t$. The work done by the electric field is:

$$\delta W = \delta q(V_a - V_b) = \delta qV,$$

where $V$ is the potential difference between $a$ and $b$, $(V_a > V_b)$. So, the rate the work is done (i.e., the rate at which the system loses energy), i.e., the power, is:

$$P = \frac{\delta W}{\delta t} = V \frac{\delta q}{\delta t} = VI \quad \text{(Watts.)}$$

Also, using Ohm’s Law: $P = I^2R = \frac{V^2}{R}$.

Electrical power and energy:

**UNITS:** $P \Rightarrow$ Joules/s $\Rightarrow$ watts (W).

$$1kW = 1000W$$

$$P = VI = I^2R = \frac{V^2}{R} \quad V = (V_a - V_b)$$

For charges to flow for $t$ seconds, the energy required is:

$$U = VIt = I^2Rt = \frac{V^2}{R}t \quad \text{(Joules),}$$

which is often dissipated in the form of *heat* or *light*, e.g.,

- Hairdryer: 1500W
- Light bulbs: 25W, 60W, 100W, etc.

Note: You pay FPL for the work done in moving charges around circuits, which appears as the energy produced ($\Rightarrow U = P \times t$).
Rating: 60W at 120V

Since $P = \frac{V^2}{R}$, the resistance of the filament in the bulb is

$$R = \frac{V^2}{P} = \frac{120^2}{60} = 240 \Omega.$$  

Note: if a potential difference of only 100V is used, the resistance of the filament, $R$, doesn’t change, but the power does:

$$P = \frac{V^2}{R} = \frac{100^2}{240} = 41.7 \text{ W},$$

i.e., the power dissipated is less so bulb is not as bright.

Examples of simple electrical circuits ...

Note: a lemon with copper and zinc nails acts as a battery! Each circuit has a battery and a load (bulb or motor). The batteries provide the potential difference to "move" charges through the load.
Another example of a simple circuit; a 0.47Ω resistor is connected across the terminals of a 9V battery. The battery provides the potential difference to move +ve charge from the

+ve terminal → −ve terminal
(high potential) (low potential)

through the resistor (load). So, the direction of the current is from high potential to low potential.

Note: from Ohm’s Law:

\[ I = \frac{V}{R} = \frac{9}{0.47} = 19.1\, \text{A} \quad \text{and} \quad P = I^2R = 172\, \text{W}. \]

The resistor gets very hot!!

More on simple circuits ...

Flashlight with two batteries

When the switch is closed, the batteries cause charges to flow through the LOAD (a bulb) and around the circuit. When the switch is opened, the charges can no longer flow around the circuit.

A generic simple circuit:

Note: current does not get “used up” in a circuit. It is the same all the way round even in the LOAD!
Consider a battery not in a circuit (i.e., not delivering a current). If the potential at a is $V_a$ and the potential at b is $V_b$ (note $V_a > V_b$) then the potential difference is

$$V_a - V_b = E,$$

and E is called the **electromotive force** (emf) of the battery.

Consider a battery in a circuit. Typically, the connecting wires a-c and b-d have negligible resistance compared with the load (R), so:

$$\Delta V_{ac} (= IR_{ac}) \Rightarrow 0 \text{ and } \Delta V_{db} (= IR_{db}) \Rightarrow 0.$$

$$\therefore V_a = V_c \text{ and } V_d = V_b$$

so the “full” emf is across the load.

**Example:** Consider a flashlight. The resistance of a bulb is $\sim 5\Omega$; the resistance of the connections between the bulb and the battery is $< 10^{-3}\Omega$.

The source of emf (i.e., the battery) is like an elevator doing work to “lift” charge from a position of low potential (−ve), $V_b$, to a position of high potential (+ve), $V_a$, where:

$$E = V_a - V_b.$$

In a mechanical analog, marbles (charges) are “raised” to a point of higher potential energy $V_c$ at c. They fall from the top of the track, to a position of lower potential $V_d$ at d, losing potential energy. At the bottom, work is done to “lift” them back to the point of high potential.
Mechanical analog of a battery:

- A battery acts like a “charge elevator” doing work to “move” charges from the −ve to the +ve terminal and raising their potential energy.
- The potential difference ($V$) is a fixed quantity so the battery “raises” each charge (δ$q$) the same amount of potential energy ($\delta U = \delta q \cdot V = \delta W$). The total power required is: $P = \frac{\delta W}{\delta t} = \frac{\delta}{\delta t}(n\delta qV) = VI$

*work done by the battery*

- The amount of current supplied, i.e., the rate of flow of charge depends on the rate the elevator moves; faster for more current, slower for less current. The “height” doesn’t change, so $V$ remains constant even though the current changes.

**Question 25.2:** Find the power dissipated in a resistor connected across a constant potential difference of 120V if its resistance is $5\Omega$. 

\[ P = \frac{\delta W}{\delta t} = \frac{\delta}{\delta t}(n\delta qV) = VI \]
Since the emf of the battery is applied across the load resistor, the current is 
\[ I = \frac{V}{R} = \frac{E}{R} = \frac{120 \text{ V}}{5 \Omega} = 24.0 \text{ A}. \]

The power dissipated by the 5Ω resistor is: 
\[ P = I^2 R = (24.0 \text{ A})^2 \times 5 \Omega = 2880 \text{ W} \Rightarrow 2.88 \text{ kW}. \]

What is the power dissipated by the battery? 
\[ P = E \times I \Rightarrow (120 \text{ V}) \times (24.0 \text{ A}) = 2.88 \text{ kW}. \]

Also, since the “full” emf of the battery is across the resistor: 
\[ P = \frac{E^2}{R} = \frac{(120 \text{ V})^2}{5 \Omega} = 2.88 \text{ kW}. \]

Let’s look at the potential around the circuit: 
We’ll start at c and move clockwise

Note that the current is constant around the circuit: 
\[ I = \frac{E}{R} = \frac{120 \text{ V}}{5 \Omega} = 24.0 \text{ A}. \]
**Question 25.3:** When an electric current passes through a load, the charges lose energy producing thermal energy in the load. Do the charges that make up the current lose (a) kinetic energy, (b) potential energy, or (c) a combination of the two?

Charges move from a point of high potential (+) to low potential (−), so their potential energy changes (δU = δq.V). But the kinetic energy of the charges does not change! If it did, it would mean that their speed changes. If the speed *decreased* the charges would build up in some region ... if the speed *increased* the charges would be “s_t_r_e_t_c_h_e_d” out! Neither of these things happen because the rate of flow of charge in a circuit (i.e., the current) is always the same everywhere ... therefore, there can be no change in speed. So, the answer is (b), charges lose potential energy only.
**QUESTION:** what happens when we connect different loads (resistors) together? There are two ways they can be connected.

- Example of resistors **IN SERIES**.

- Examples of resistors **IN PARALLEL**.

Resistances **IN SERIES**:

The current through each resistor, R₁, R₂, and R₃, is the same. An equivalent resistor will produce the same potential difference (V) for the same current (I):

\[ V = I R_{eq} = I (R_1 + R_2 + R_3) \]

\[ \therefore R_{eq} = R_1 + R_2 + R_3. \]

In general with resistors in series:

\[ R_{eq} = \sum_i R_i. \]
Resistors in series ...

Total resistance ⇒ \( R_1 + R_2 = 2\Omega + 4\Omega = 6\Omega \).

\[ I = \frac{E}{R_{\text{total}}} = \frac{12\, \text{V}}{6\, \Omega} = 2.0\, \text{A}. \]

Potential difference across \( R_1 \) ⇒ \( V_1 = IR_1 = 4 \) volts.

Potential difference across \( R_2 \) ⇒ \( V_2 = IR_2 = 8 \) volts.

Note: \( V_1 + V_2 = 12 \) volts = \( E \).

Look at the potential difference around the circuit:

Unfortunately, real batteries come with “internal resistance” ... as in this example

**Question 25.4**: An 11.0Ω resistor is connected across a battery with an emf of 6.00 V and internal resistance 1.00Ω. Find:

(a) the current in the circuit,
(b) the voltage across the terminals of the battery,
(c) the power supplied by the battery,
(d) the power delivered to the external resistor, and
(e) the power delivered to the battery’s internal resistance.
Note the resistors are in series ...

(a) The total resistance in this circuit is:
\[ R = 11\,\Omega + 1\,\Omega = 12\,\Omega. \]
\[ \therefore I = \frac{E}{R} = \frac{6\,\text{V}}{12\,\Omega} = 0.5\,\text{A}. \]
(With no internal resistance \( R = 11\,\Omega \) and \( I \Rightarrow 0.55\,\text{A} \).)

(b) The potential difference across battery terminals:
\[ V = E - Ir = (6\,\text{V}) - (0.5\,\text{A} \times 1\,\Omega) = 5.5\,\text{V}. \]
Note that this is less than the emf ... the emf of a battery is the potential difference across the battery terminals with no load, i.e., when it not supplying current.

(c) \( P = EI = (6\,\text{V}) \times (0.5\,\text{A}) = 3.0\,\text{W}. \)

(d) \( P_R = I^2R = (0.5\,\text{A})^2 \times 11\,\Omega = 2.75\,\text{W}. \)

(e) \( P_r = I^2r = (0.5\,\text{A})^2 \times 1\,\Omega = 0.25\,\text{W}. \)

**Question 25.5:** A battery has an emf equal to 12.0 V and an internal resistance of 4.00 Ω. What value of external resistance \( R \) should be placed across the terminals of the battery to obtain maximum power delivered to the resistor?
The power delivered to the resistor is $P = I^2 R$, where

$$I = \frac{E}{R + r}, \quad \text{i.e.,} \quad P = \frac{E^2 R}{(R + r)^2}.$$ 

For maximum power, $\frac{dP}{dR} = 0$.

$$\frac{dP}{dR} = \frac{(R + r)^2 E^2 - 2E^2 R(R + r)}{(R + r)^4} = \frac{(r - R)E^2}{(R + r)^3}.$$ 

Hence, $\frac{dP}{dR} = 0$ when $R = r = 4.00\Omega$. This is the condition for maximum power. Also true in ac circuits.

Resistors *IN PARALLEL*:

Note: *the potential difference across each resistor* ($V_i = I_i R_i$) *is the same* but if $R_1 \neq R_2 \neq R_3$ the current through each resistor will be different. However,

$$I = I_1 + I_2 + I_3. \quad (This \ Kirchoff's 1st rule ... later.)$$

$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_{eq}} = V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right),$$

i.e.,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$ 

In general, with resistors in parallel,

$$\frac{1}{R_{eq}} = \sum \frac{1}{R_i}.$$
**Question 25.6:** If the bulbs and batteries are identical, in which circuit are the bulbs brighter or are they both the same?

The brightness of a bulb is determined by the power dissipated; the more power, the brighter the bulb.

In (a) the current through each bulb is \( I_a = \frac{E}{2R} \).

Therefore, the power dissipated by each bulb in (a) is:

\[
P_a = \left(\frac{E}{2R}\right)^2 R = \frac{E^2}{4R}.
\]

In (b) since the potential difference across the bulbs is the same, the current through each bulb is \( I_b = \frac{E}{R} \).

Therefore, the power dissipated by each bulb in (b) is:

\[
P_b = \left(\frac{E}{R}\right)^2 R = \frac{E^2}{R}.
\]

\[\therefore P_b = 4P_a,\]

so the bulbs are brighter in (b).
**Question 25.7:** If the bulbs and batteries are identical, which circuit draws most power from the battery?

Assume the batteries have zero internal resistance.

Let the resistance of each bulb be \( r \). If \( E \) is the emf of each battery, the power dissipated in each circuit is

\[
\frac{E^2}{R_{\text{tot}}},
\]

where \( R_{\text{tot}} \) is the total equivalent resistance.

- In (a): \( R_{\text{tot}} = r \).
- In (b):

\[
\frac{1}{R_{\text{tot}}} = \frac{1}{(r + r)} + \frac{1}{(r + r)} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r}.
\]

i.e., \( R_{\text{tot}} = r \).

- In (c):

\[
\frac{1}{R_{\text{tot}}} = \frac{1}{3r} + \frac{1}{3r} + \frac{1}{3r} = \frac{3}{3r} = \frac{1}{r}.
\]

i.e., \( R_{\text{tot}} = r \).

Since the equivalent resistances of (a), (b) and (c) are the same, the power drawn from the batteries (and the currents) is the same in each case!
**Question 25.8**: If the battery in the circuit shown has negligible internal resistance, find

(a) the current in each resistor, and
(b) the power delivered by the battery.

![Circuit Diagram]

(a) First, we calculate $I_1$, but to do that we need the total resistance of the circuit. We recognize that the right hand group of 3 resistors are all in parallel, so the equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{2\Omega} + \frac{1}{2\Omega} + \frac{1}{4\Omega} = \frac{2 + 2 + 1}{4\Omega} = \frac{5}{4\Omega}.$$  

∴ $R_{eq} = \frac{4}{5\Omega} = 0.8\Omega$.

So, here is the equivalent circuit:

![Equivalent Circuit Diagram]

The $3\Omega$ resistor is in series with the equivalent resistance of $0.8\Omega$, so

$$I_1 = \frac{6 \text{ V}}{(3\Omega + 0.8\Omega)} = 1.58 \text{ A.}$$
To determine the currents $I_2$, $I_3$ and $I_4$ through the parallel combination we need to find the potential difference across them, i.e., $V_{ab}$. By Ohm’s Law:

$$V_{ab} = 1.58 \, \text{A} \times 0.8 \, \Omega = 1.26 \, \text{V}.$$ 

The potential difference across each of the three resistors (in parallel) is the same:

$$\therefore I_2 = 1.26 \, \text{V} \div 2\Omega = 0.63 \, \text{A}; \quad I_3 = 1.26 \, \text{V} \div 4\Omega = 0.32 \, \text{A}; \quad I_4 = 1.26 \, \text{V} \div 2\Omega = 0.63 \, \text{A}.$$ 

(Note that: $I_1 = I_2 + I_3 + I_4 = 1.58 \, \text{A}$, which it must since current cannot be lost or gained.)

(b) The power delivered by the battery: $P = E I_1$, where $E$ is the emf of the battery.

$$\therefore P = 6 \, \text{V} \times 1.58 \, \text{A} = 9.48 \, \text{W}.$$ 

Also, $P = \frac{E^2}{R_{\text{tot}}}$, where $R_{\text{tot}}$ is the equivalent resistance of the ensemble of resistors. From (a)

$$R_{\text{tot}} = 3\Omega + 0.8\Omega = 3.8\Omega.$$ 

$$\therefore P = \frac{(6 \, \text{V})^2}{3.8\Omega} = 9.47 \, \text{W}.$$ 

(Rounding error.)

You could also calculate the power dissipated by each of the four resistors, i.e., $P = \sum I_i^2 R_i$ but it takes more time!
**Question 25.9**: Four 4Ω resistors are arranged as shown.

(a) What is the equivalent resistance between \(a\) and \(b\)?

(b) What is the equivalent resistance between \(a\) and \(b\) if an additional 4Ω resistor was connected across \(c\) and \(d\)?

The circuit is equivalent to two 8Ω resistors in parallel.

\[
\frac{1}{R_{eq}} = \frac{1}{8\Omega} + \frac{1}{8\Omega} = \frac{1}{4\Omega},
\]

i.e., \(R_{eq} = R_{ab} = 4\Omega\).
If we put a battery across \(a-b\), equal currents will flow through \(c\) as through \(d\) because the two “routes” are identical (a resistance of \(8\,\Omega\)). Since the resistors are all the same (\(4\,\Omega\), the potential at \(c\) is the same as the potential at \(d\)!

Therefore, there’s no potential difference between \(c\) and \(d\), so no current will flow through any resistor connected across \(c-d\). As no “extra” current is required from the battery, the equivalent resistance remains the same!

With more complex circuits, e.g., with more than one source of emf, we must follow certain rules (which we’ve already come across) ...

**Kirchhoff’s Rules:**

[1] The algebraic sum of currents at a junction is zero, i.e., when currents meet at a junction \(\sum I_i = 0\).

\[ I_1 + I_2 + I_3 - I_4 - I_5 = 0, \]

i.e., \(I_1 + I_2 + I_3 = I_4 + I_5\).
The algebraic sum of the potential differences around a circuit is zero, i.e., around a circuit $\Sigma V_i = 0$.

Example ...

Example diagram

Remember $V_a > V_b$ and $V_c > V_d$, i.e., the potential difference across a resistor decreases in the direction of the current, and across the battery $V_f > V_e$.

$\therefore E - V_{ab} - V_{cd} = 0$ (Kirchoff’s 2nd rule), i.e., $V_{ab} + V_{cd} = E$.

**Question 25.10**: (a) What is the potential difference $\Delta V$ across the unknown circuit element in the direction of the current? (b) Which point, X or Y, is at the higher potential?
(a) By Kirchoff’s 2nd rule, in the direction of the current, and starting from A, we have:

\[ +12 \text{ V} + \Delta V - 6 \text{ V} - 8 \text{ V} = 0 \]

\[ \therefore \Delta V = +2 \text{ volts.} \]

(b) Since \( \Delta V > 0 \) the potential increases in the direction of the current so the potential at Y is higher than at X. So it cannot be a resistor; it is a source of emf, like a 2V battery.

---

**Question 25.11**: In the circuit shown below, the batteries have negligible internal resistance. Find (a) the current in each branch of the circuit, the potential difference between point X and Y, and (c) the power supplied by each battery.
(a) Choose the directions of the currents arbitrarily. Using Kirchhoff’s rules we have:

\[ I_1 = I_2 + I_3 \]  ...  ...  ...  ...  (1)

In circuit A going clockwise (starting at battery):

\[ +12 - 4I_1 - 6I_3 = 0, \]
i.e., \[ 4I_1 + 6I_3 = 12 \]  ...  ...  ...  ...  (2)

In circuit B we have (starting at X):

\[ -3I_2 - 12 + 6I_3 = 0 \]
i.e., \[ -3I_2 + 6I_3 = 12 \]  ...  ...  ...  ...  (3)

Using (1), substitute for \( I_1 \) in (2). Then (2) becomes:

\[ 4(I_2 + I_3) + 6I_3 = 12 \]
i.e., \[ 4I_2 + 10I_3 = 12 \]  ...  ...  ...  ...  (4)

Equations (3) and (4) are a pair of simultaneous linear equations.
Solving equations (3) and (4) as a pair of simultaneous linear equations gives:

\[ I_2 = -0.90 \text{ A} \quad \text{and} \quad I_3 = 1.56 \text{ A}. \]

(The negative sign means we chose the wrong direction for \( I_2 \)!) \[ \therefore I_1 = (-0.90 \text{ A}) + 1.56 \text{ A} = 0.66 \text{ A}. \]

(b) \( V_{XY} = I_3 \times 6\Omega = 1.56 \text{ A} \times 6\Omega = 9.36 \text{ V}. \)

(c) \( P = EI. \)

\[ \therefore P_A = 12 \text{ V} \times 0.66 \text{ A} = 7.92 \text{ W}, \]

and \( P_B = 12 \text{ V} \times 0.90 \text{ A} = 10.80 \text{ W}. \)

Simple R-C circuit:

Charging the capacitor:

Close \( S_1 \) and leave \( S_2 \) open at \( t = 0 \). Charges flow through resistor \( R \) and onto the capacitor \( C \). At some time \( t \), let the charge on the capacitor be \( q \). Using Kirchoff’s 2nd rule: \( +E - V_R - V_C = 0, \)

\[ \text{i.e.,} \quad i_1R + \frac{q}{C} - E = 0, \]

or \[ \frac{dq}{dt} + \frac{q}{RC} - \frac{E}{R} = 0. \]

This is a \textit{1st-order differential equation} that tells us how \( q \) varies with \( t \).
The solution is

\[ q(t) = EC(1 - e^{-t/\tau}) \]

where \( Q_F = EC \) is the final (maximum) charge and the quantity \( \tau = RC \) is called the \textit{TIME CONSTANT}, which gives us a measure of how quickly the capacitor becomes charged.

When \( t = \tau = RC \)

\[ q(\tau) = Q_F \left( 1 - e^{-1} \right) = 0.632Q_F. \]

Also, since \( V_C(t) = \frac{q(t)}{C} \) and \( Q_F = EC \),

then \( V_C(\tau) = \frac{0.632Q_F}{C} = 0.632E. \)

Since \( q(t) = EC(1 - e^{-t/\tau}) \), the current \( i_1 \) at any time \( t \) is:

\[ i_1(t) = \frac{dq}{dt} = \frac{E}{R} e^{-t/\tau} \]

\[ = I_o e^{-t/\tau} = I_o e^{-t/\tau}, \]

where \( I_o = \frac{E}{R} \) is the \textit{initial} current in the circuit the instant \( S_1 \) is closed.

When \( t = \tau \),

\[ i_1(\tau) = I_o e^{-1} = 0.368I_o. \]

Also \( V_R(t) = i_1(t)R \).

\[ \therefore V_R(\tau) = 0.368E. \]

\textbf{Note:} when connected across a battery,

- an \textit{uncharged} capacitor “acts” like an short-circuit so the current is a maximum \( (i_1(0) \Rightarrow \text{max} = I_o). \)
- a \textit{fully charged} capacitor “acts” like an open-circuit, i.e., an infinite resistance, so the current is zero \( (i_1(\infty) \Rightarrow \text{min} = 0). \)
Discharging the capacitor:

Now, start with a charge $Q_0$ on the capacitor and switch $S_1$ open; so the capacitor is “charged”.

Close $S_2$ at $t = 0$. The capacitor discharges so charges (i.e., a current $i_2$) flows through $R$. Then at some time $t$, we have from Kirchoff’s 2nd rule, in the direction of the arrow (and current $i_2(t)^*$):

$$V_C - V_R = 0,$$

i.e.,

$$i_2(t)R - \frac{q}{C} = 0,$$

or

$$\frac{dq}{dt} - \frac{q}{RC} = 0.$$

* Note that $i_2$ during discharging is in the opposite direction to $i_1$ during charging.

The solution is:

$$q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}.$$

When $t = \tau$:

$$q(\tau) = Q_0 e^{-\tau/\tau} = Q_0 e^{-1} = 0.368Q_0.$$

Also, since $V_C(t) = \frac{q(t)}{C}$,

then $V_C(\tau) = \frac{0.368Q_0}{C}$. 
Since \( q(t) = Q_0 e^{-t/RC} \), the current at any time \( t \) is:
\[
i_2(t) = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_o e^{-t/\tau},
\]
where \( I_o = -\frac{Q_0}{RC} \), i.e., the initial current at \( t = 0 \) when \( S_2 \) is closed.

When \( t = \tau \):
\[
i_2(\tau) = I_o e^{-\tau/\tau} = \frac{Q_0}{RC} e^{-1} = 0.368 \frac{Q_0}{RC}.
\]
Also, since \( V_R(t) = i_2(t)R \),

then \( V_R(\tau) = \frac{0.368Q_0}{C} = V_C(\tau) \).

During the charging process we have ...
\[
E - V_R - V_C = 0
\]
i.e., \( V_R + V_C = E \).

During the discharging process we have ...
\[
V_C - V_R = 0
\]
i.e., \( V_R = V_C \).
**Question 25.12:** In the circuit shown below, switch S is opened after having been closed for a long time so the capacitor is fully discharged. Four seconds after S is opened, the potential difference across the resistor is 20.0 V. What is the resistance of the resistor?

With switch S closed, the capacitor is completely uncharged and the potential difference across R is 50 V. When switch S is opened at \( t = 0 \), the capacitor begins to charge and the potential difference across R decreases. When \( t = 4 \text{s} \), \( V_R = 20 \text{ V} \), what is the value of R?

We know: \( V_R \Rightarrow V_R(t) = i(t)R = I_o \exp(-t/RC) \).

But initially: \( V_R(0) = I_o R = E \).

\[ \therefore V_R(t) = E \exp(-t/RC) \]

i.e., \( \exp(-t/RC) = \frac{V_R(t)}{E} \).

Take natural logarithms of both sides...

\[ -\frac{t}{RC} = \ln\left(\frac{V_R(t)}{E}\right) \]

\[ \therefore -\frac{4\text{s}}{(2 \times 10^{-6} \text{F}) \times R} = \ln\left(\frac{20 \text{ V}}{50 \text{ V}}\right) = -0.916 \]

i.e., \( R = \frac{4\text{s}}{(2 \times 10^{-6} \text{F}) \times 0.916} = 2.18 \times 10^6 \Omega \).

The time constant is \( \tau = RC = 4.37 \text{s} \).
During charging, a current flows through the battery and resistor R. The instantaneous power dissipated by the battery is $E \times i(t)$.

∴ Total energy dissipated by the battery to fully charge the capacitor is

$$E \int_{0}^{\infty} i(t) \, dt = E \int_{0}^{\infty} \frac{dq}{dt} \, dt = E \int_{0}^{Q_F} dq = Q_F E.$$  

But, when the capacitor is charged to $Q_F$, the energy stored in the capacitor is $\frac{1}{2} Q_F E$. So, using conservation of energy, the energy dissipated by the resistor is:

$$Q_F E - \frac{1}{2} Q_F E \Rightarrow \frac{1}{2} Q_F E.$$  

Note, if the instantaneous current is $i(t)$, the instantaneous power dissipated by R is:

$$P_R(t) = i^2(t) R,$$

and the instantaneous power output of the battery is $Ei(t)$.

So, the energy stored in the capacitor at time $t$ is

$$U_C(t) = Ei(t) - i^2(t) R.$$